Scattering of a Plane Wave on a Ferrite Cylinder at Normal Incidence*

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Summary—The scattered field is given as a series of cylinder functions. If the ferrite cylinder is magnetized along its axis the scattering pattern becomes asymmetrical about the direction of incidence. Approximation formulas for the thin cylinder and the far field zone are given. It is shown that in the first approximation the amplitude is an even function and the phase angle of the field is an odd function of the scattering angle. Exact numerical results have been obtained with a Univac digital computer. By a suitable arrangement of the ferrite cylinders, a unidirectional pattern can be obtained which is controlled by the applied magnetic dc field.

Introduction

The scattering of a plane wave on a dielectric cylinder has been discussed in a number of papers. The complete solution for arbitrary incidence has been given by Wait.[1] He also discussed approximate solutions for the far field zone and the case of a cylinder whose diameter is small compared with the wavelength.

In this paper the scattering of a plane wave from a homogeneous ferrite cylinder is investigated. The discussion is restricted to the case of normal incidence, for which an exact solution in form of a series of Bessel functions can be found. The ferrite rod is magnetized along its axis. Because of the nonreciprocal properties of the ferrite material, a nonsymmetrical distribution of the scattered field with respect to the direction of incidence is to be expected. The scattered field is a function of the permeability tensor of the ferrite, which in turn depends on the applied magnetic dc field. It is, therefore, possible to control the scattering pattern of the ferrite cylinder by the magnetic field.

The Mathematical Solution

An infinitely long ferrite cylinder with its axis along the z direction is considered. A dc magnetic field is applied along its axis. A plane wave is incident in the positive x direction. With these assumptions the problem is reduced to two dimensions in the x-y plane. The polarization of the wave is arbitrary. The field can then be decomposed in two waves, one which is polarized normal to the cylinder axis ($E=E_{\alpha y}$) and one which is polarized parallel to it ($E=E_{\alpha z}$) (see Fig. 1). In the first case the magnetic field of the wave is parallel to the axis and to the applied magnetic dc field and, therefore, no nonreciprocal interaction between the field and the magnetization of the ferrite takes place. Hence the tensor permeability reduces to a scalar. In other words the problem is the same as the one for a dielectric cylinder.[1] For the wave polarized parallel to the z axis, however, the magnetic dc and ac fields are normal to each other, so that the ac field interacts with the precessing magnetic dipoles of the ferrite. In this case one has to use the tensor permeabilities in Maxwell's equations,

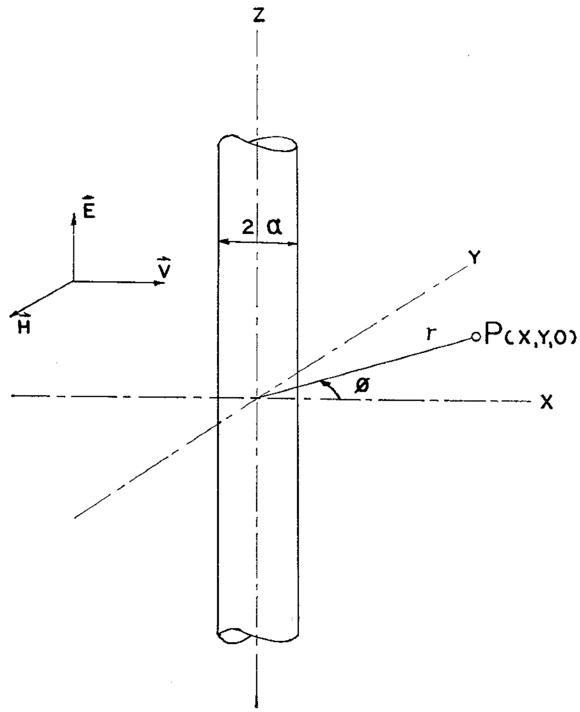

Fig. 1—Plane wave incident normally on a circular ferrite cylinder.

$$\text{curl}\, \boldsymbol{H} = \frac{\partial \boldsymbol{D}}{\partial t} = j\omega D = j\omega\epsilon \boldsymbol{E}, \tag{1a}$$

$$\text{curl}\, \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} = -j\omega \boldsymbol{B} = -j\omega\bar{\mu}\cdot \boldsymbol{H}. \tag{1b}$$

Because there is no variation in the z direction, (1b) can be replaced by the two-dimensional tensor equation

$$\boldsymbol{B} = -\frac{1}{j\omega}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\cdot \boldsymbol{\nabla} E_z = -\frac{1}{j\omega}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\cdot \begin{pmatrix} \dfrac{\partial}{\partial x} \\ \dfrac{\partial}{\partial y} \end{pmatrix} E_z, \tag{2}$$

where $\boldsymbol{\nabla}$ is the two-dimensional symbolic gradient. Thus the magnetic field $\boldsymbol{H}$ is in free space

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[1] J. R. Wait, "Scattering of a plane wave from a circular dielectric cylinder at oblique incidence," *Can. J. Phys.*, vol. 33, pp. 189–195; 1955.

$$H = -\frac{1}{j\omega}\mu_0^{-1}\cdot\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\cdot\nabla E_z, \tag{3}$$

in ferrite

$$H = -\frac{1}{j\omega}(\mu)^{-1}\cdot\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\cdot\nabla E_z. \tag{4}$$

The permeability tensor $\bar{\mu}$ can be calculated from the simple model of the precessing magnetic dipole moment of the electron in a magnetic field.[2] One obtains in two dimensions

$$\bar{\mu} = \begin{pmatrix} \mu & -jk \\ jk & \mu \end{pmatrix}, \tag{5}$$

where the two components μ and k are given by

$$\mu = \mu_0 \frac{\gamma^2\mu_0 H_0 B_z - \omega^2}{\gamma^2\mu_0^2 H_0^2 - \omega^2} \tag{6}$$

and

$$k = \mu_0 \frac{\omega\gamma\mu_0 M_z}{\gamma^2\mu_0^2 H_0^2 - \omega^2}, \tag{7}$$

where $B_z = \mu_0(H_0 + M_z)$ and γ = gyromagnetic ratio of the electron spin. From (5) the reciprocal tensor permeability can be calculated

$$(\bar{\mu})^{-1} = \frac{1}{\mu^2 - k^2}\begin{pmatrix} \mu & jk \\ -jk & \mu \end{pmatrix}. \tag{8}$$

Combining (4) and (8),

$$H = -\frac{1}{j\omega}\frac{1}{\mu^2 - k^2}\begin{pmatrix} -jk & \mu \\ -\mu & -jk \end{pmatrix}\cdot\nabla E_z. \tag{9}$$

This gives in Cartesian coordinates

$$H_x = \frac{1}{j\omega(\mu^2 - k^2)}\left(jk\frac{\partial E_z}{\partial x} - \mu\frac{\partial E_z}{\partial y}\right) \tag{10a}$$

$$H_y = \frac{1}{j\omega(\mu^2 - k^2)}\left(\mu\frac{\partial E_z}{\partial x} + jk\frac{\partial E_z}{\partial y}\right), \tag{10b}$$

and in cylindrical coordinates

$$H_r = \frac{1}{\omega(\mu^2 - k^2)}\left(k\frac{\partial E_z}{\partial r} + j\mu\frac{1}{r}\frac{\partial E_z}{\partial \phi}\right) \tag{11a}$$

$$H_\phi = \frac{1}{\omega(\mu^2 - k^2)}\left(j\mu\frac{\partial E_z}{\partial r} - k\frac{1}{r}\frac{\partial E_z}{\partial \phi}\right). \tag{11b}$$

Incident Field

In order to apply the boundary conditions (continuity of the tangential electric and magnetic field at $r=a$) it is necessary to express the incoming plane wave in cylindrical coordinates. This can be done in terms of a series of Bessel functions[3]

$$E_z^{\text{inc}} = E_0 e^{j(\omega t - \beta x)} = E_0 e^{j(\omega t - \beta r\cos\phi)}$$
$$= E_0 e^{j\omega t}\sum_{n=-\infty}^{\infty}(j)^n J_n(\beta r)e^{-jn\phi}. \tag{12}$$

The magnetic field is given by (11) when $\mu=\mu_0$ and $k=0$.

$$H_r^{\text{inc}} = \frac{1}{\omega\mu_0}\cdot\frac{1}{r}\frac{\partial E_z}{\partial \phi} = \frac{E_0}{\omega\mu_0 r}\sum_{n=-\infty}^{\infty}(j)^n J_n(\beta r)ne^{-jn\phi} \tag{13a}$$

$$H_\phi^{\text{inc}} = \frac{j}{\omega\mu_0}\frac{\partial E_z}{\partial r} = \frac{j\beta E_0}{\omega\mu_0}\sum_{n=-\infty}^{\infty}(j)^n J_n'(\beta r)e^{-jn\phi}. \tag{13b}$$

All derivatives are with respect to the argument βr.

Scattered Field

For the scattered field a similar representation is chosen. Here the Hankel functions of the second kind have to be used, because the asymptotic approximation for large arguments give a decreasing outgoing wave.

$$E_z^{\text{scat}} = \sum_{n=-\infty}^{\infty} a_n^s H_n^{(2)}(\beta r)e^{-jn\phi} \tag{14}$$

$$H_r^{\text{scat}} = \frac{1}{\omega\mu_0}\frac{1}{r}\sum_{n=-\infty}^{\infty} a_n^s H_n^{(2)}(\beta r)ne^{-jn\phi} \tag{15a}$$

$$H_\phi^{\text{scat}} = \frac{j\beta}{\omega\mu_0}\sum_{n=-\infty}^{\infty} a_n^s H_n'^{(2)}(\beta r)e^{-jn\phi}. \tag{15b}$$

Inside Field

Inside the ferrite the field is represented by Bessel functions of the first kind.

$$E_z = \sum_{n=-\infty}^{\infty} a_n J_n(\beta_2 r)e^{-jn\phi} \tag{16}$$

$$H_r = \frac{1}{\omega(\mu^2 - k^2)}\left[k\beta_2\sum_{n=-\infty}^{\infty} a_n J_n'(\beta_2 r)e^{-jn\phi} + \frac{\mu}{r}\sum_{n=-\infty}^{\infty} a_n J_n(\beta_2 r)ne^{-jn\phi}\right] \tag{17a}$$

$$H_\phi = \frac{j}{\omega(\mu^2 - k^2)}\left[\mu\beta_2\sum_{n=-\infty}^{\infty} a_n J_n'(\beta_2 r)e^{-jn\phi} + \frac{k}{r}\sum_{n=-\infty}^{\infty} a_n J_n(\beta_2 r)_n e^{-jn\phi}\right]. \tag{17b}$$

It can be shown that in the ferrite material the wave number is given by

$$\beta_2^2 = \omega^2\mu_{\text{eff}}\cdot\epsilon, \tag{18}$$

where

$$\mu_{\text{eff}} = \frac{\mu^2 - k^2}{\mu}. \tag{19}$$

In (13) to (17) the harmonic time dependence $e^{j\omega t}$ is omitted.

[2] C. L. Hogan, "The microwave gyrator," *Bell Sys. Tech. J.*, vol. 31, pp. 1–31; January, 1952.
[3] J. H. Stratton, "Electromagnetic Theory," McGraw-Hill Book Co., Inc., New York, N. Y., p. 372; 1941.

$$E_z^s(r,\phi) = \frac{1}{4}\pi(\beta a)^2 E_0 \left[-j\left(\frac{\epsilon}{\epsilon_0} - 1\right) H_0^{(2)}(\beta r) + H_1^{(2)}(\beta r) \left\{ \frac{\frac{\mu_0}{\mu_{eff}}\left(1+\frac{k}{\mu}\right) - 1}{\frac{\mu_0}{\mu_{eff}}\left(1+\frac{k}{\mu}\right) + 1} e^{-j\phi} - \frac{\frac{\mu_0}{\mu_{eff}}\left(1-\frac{k}{\mu}\right) - 1}{\frac{\mu_0}{\mu_{eff}}\left(1-\frac{k}{\mu}\right) + 1} e^{j\phi} \right\} \right]. \quad (27)$$

Boundary Conditions

Continuity of the tangential electric and magnetic field requires

$$E_z^{inc} + E_z^{scat} = E_z \text{ at } r = a \quad (20)$$

$$H_\phi^{inc} + H_\phi^{scat} = H_\phi \text{ at } r = a. \quad (21)$$

Substituting (12) to (17) in (20) and (21) and eliminating a_n yields

$$a_n^s = -E_0(j)^n \frac{\frac{D_n(\beta_2 a)}{J_n(\beta_2 a)} - \frac{J_n'(\beta a)}{J_n(\beta a)}}{\frac{D_n(\beta_2 a)}{J_n(\beta_2 a)} - \frac{H_n'^{(2)}(\beta a)}{H_n^{(2)}(\beta a)}} \frac{J_n(\beta a)}{H_n^{(2)}(\beta a)}, \quad (22)$$

where

$$D_n(\beta_2 a) = \frac{\mu_0}{\mu_{eff}} \frac{\beta_2}{\beta}\left[J_n'(\beta_2 a) + \frac{k}{\mu}\frac{n}{\beta_2 a} J_n(\beta_2 a)\right]. \quad (23)$$

For $k \to 0$, the permeability tensor $\bar{\mu}$ reduces to a scalar and one obtains the case of the dielectric cylinder.[1]

B. Far Field Approximation, $\beta r \gg 1$

Here the asymptotic approximation for the Hankel function with large argument is used

$$H_n^{(2)}(z) \underset{z\to\infty}{\simeq} \sqrt{\frac{2}{\pi z}}\, e^{-jz+j(n\pi/2)+j(\pi/4)} = (j)^n H_0^{(2)}(z), \quad (28)$$

where $H_0^{(2)}(z)$ is given by this equation. Combining (14), (22), and (28) gives

$$E_z^s = jH_0^{(2)}(\beta r)\left[c_0 + \sum_{n=1}^{\infty} (-1)^n (c_{-n} + c_n) \cos n\phi + j(c_{-n} - c_n) \sin n\phi \right], \quad (29)$$

where

$$c_{\pm n} = -(j)^{1-n} a_{\pm n}^s. \quad (30)$$

The coefficients $c_{\pm n}$ can be expressed in terms of cylinder functions of positive order,

$$c_{\pm n} = +j \left\{ \frac{\frac{\epsilon}{\epsilon_0}\frac{\beta}{\beta_2}\left[\frac{n}{\beta_2 a}\left(1 \pm \frac{k}{\mu}\right) - \frac{J_{n+1}(\beta_2 a)}{J_n(\beta_2 a)}\right] - \frac{n}{\beta a} + \frac{J_{n+1}(\beta a)}{J_n(\beta a)}}{\frac{\epsilon}{\epsilon_0}\frac{\beta}{\beta_2}\left[\frac{n}{\beta_2 a}\left(1 \pm \frac{k}{\mu}\right) - \frac{J_{n+1}(\beta_2 a)}{J_n(\beta_2 a)}\right] - \frac{n}{\beta a} + \frac{H_{n+1}^{(2)}(\beta a)}{H_n^{(2)}(\beta a)}} \right\} \frac{J_n(\beta a)}{H_n^{(2)}(\beta a)} E_0. \quad (31)$$

Approximation Formulas

A. Thin Cylinder, $\beta a \ll 1$, $\beta_2 a \ll 1$

It is assumed that the wavelength is much larger than the radius of the cylinder. Then the cylinder functions can be expanded in power series and by keeping only the the first terms (22) becomes

$$a_0^s = -\frac{1}{4}\pi(\beta a)^2\left(\frac{\epsilon}{\epsilon_0} - 1\right) E_0 j \quad (24)$$

$$a_{\pm n}^s = \pm \frac{n}{2^{2n}(n!)^2}\pi(\beta a)^{2n} \frac{\frac{\mu_0}{\mu_{eff}}\left(1 \pm \frac{k}{\mu}\right) - 1}{\frac{\mu_0}{\mu_{eff}}\left(1 \pm \frac{k}{\mu}\right) + 1} E_0 (j)^{n+1}. \quad (25)$$

For the dielectric case ($k=0$),

$$a_n^s = -a_{-n}^s. \quad (26)$$

In the first approximation all terms except the ones with $n=0$, ± 1 can be neglected,

For the dielectric case one obtains $c_n = c_{-n}$ and the sine term in (29) vanishes. This results in a symmetrical pattern with respect to $\phi=0$. If the approximation for the thin cylinder is used, (25) and (30) yield

$$c_{\pm n} = \pm \frac{n}{2^{2n}(n!)^2}\pi(\beta a)^{2n} \frac{\frac{\mu_0}{\mu_{eff}}\left(1 \pm \frac{k}{\mu}\right) - 1}{\frac{\mu_0}{\mu_{eff}}\left(1 \pm \frac{k}{\mu}\right) + 1} E_0 \quad (32)$$

so that the $c_{\pm n}$'s are real. It can be easily shown that the amplitude of $E_z(r, \phi)$ is an even function of ϕ. In order to obtain an asymmetrical pattern the $c_{\pm n}$'s have to be complex, that is, the second-order approximations of the cylinder functions have to be taken in (22). Hence, an almost symmetrical pattern of the field strength for a cylinder for which $a \ll 1/\beta_2 = \lambda^2/2\pi$ is to be expected. In the X-band region the wavelength is about 3 cm in free space and 1 cm in ferrite. That requires $\ll 1.5$ mm.

It is interesting to note, however, that the phase angle θ of the field is an odd function of the scattering angle ϕ in the thin cylinder approximation. From (29) one obtains

$$\tan\theta = \frac{\sum_{n=1}^{\infty}(-)^n(c_{-n}-c_n)\sin\phi}{c_0+\sum_{n=1}^{\infty}(-1)^n(c_{-n}+c_n)\cos\phi}$$

$$\cong \frac{\left(\frac{\mu_0}{\mu_{eff}}\right)^2\frac{k}{\mu}\sin\phi}{\left[\left(\frac{\mu_0}{\mu_{eff}}\right)^2\left(1+\left(\frac{k}{\mu}\right)^2\right)-1\right]\cos\phi-\frac{1}{2}\left(\frac{\epsilon}{\epsilon_0}-1\right)\left[\left(\frac{\mu_0}{\mu_{eff}}+1\right)^2-\left(\frac{k}{\mu}\frac{\mu_0}{\mu_{eff}}\right)^2\right]} = \frac{a\sin\phi}{b\cos\phi-c}, \quad (33)$$

where a, b, and c are defined by this equation. The maximum phase difference is given by $d/d\phi(\tan\theta)=0$, which yields a deflection angle

$$\phi_{max} = \cos^{-1}\frac{b}{c}\,. \quad (34)$$

Numerical Evaluation

Eq. (33) shows that the phase angle of the scattered wave does not depend on the wavelength or the cylinder radius in the first approximation for the thin cylinder. To get an idea of the order of magnitude of phaseshift, let us consider some typical values for the parameters μ_0/μ_{eff} and ϵ/ϵ_0.

$$\frac{\mu_0}{\mu_{eff}} = 1, \qquad \frac{\epsilon}{\epsilon_0} = 11.$$

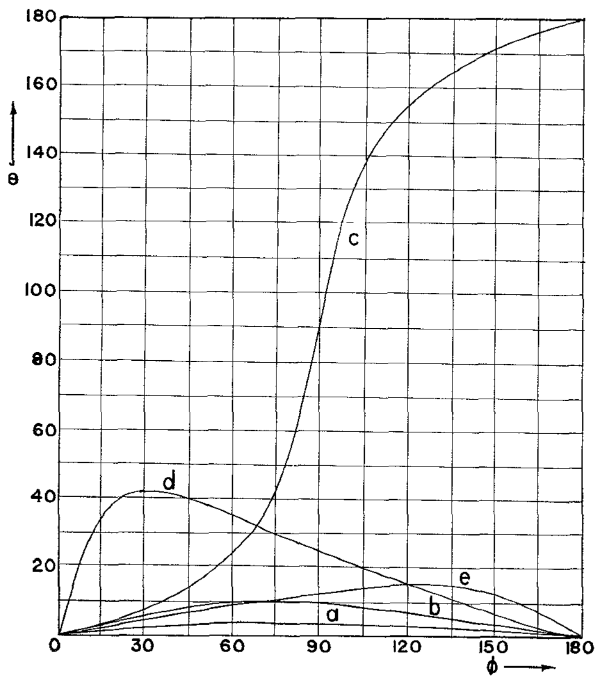

Fig. 2—Phase angle θ of scattered wave as a function of scattering angle ϕ for thin cylinder far field approximation. (a) $k/\mu=1$; (b) $k/\mu=1.5$; (c) $k/\mu=1.8$; (d) $k/\mu=2$; (e) $k/\mu=2.5$.

Then (33) becomes

$$\tan\theta = \frac{\frac{k}{\mu}\sin\phi}{\left(\frac{k}{\mu}\right)^2\cos\phi+5\left(\frac{k}{\mu}\right)^2-20}\,. \quad (35)$$

Fig. 2 gives $\theta(\phi)$ for various values of k/μ. For small and large values of k/μ the largest phaseshift is at a deflection angle $\phi\sim 90°$. For values $1.83<k/\mu<2.24$, where $\tan\theta$ becomes infinite at certain deflection angles, the phase angle θ is steadily increasing in the positive direction of ϕ resulting in a spiral wave (Fig. 3). This result can be explained by means of the precessing magnetic dipoles. From (33) it can be seen that for a nondielectric cylinder ($\epsilon/\epsilon_0=1$) the scattered wave is spiral for all values of the magnetic field (*e.g.* for all values of $k/\mu\neq 0$). For high values of ϵ/ϵ_0 and low values of k/μ, however, the dielectric properities of the ferrite are predominant, and the spiral wave is "covered up" by the wave scattered on a dielectric cylinder.

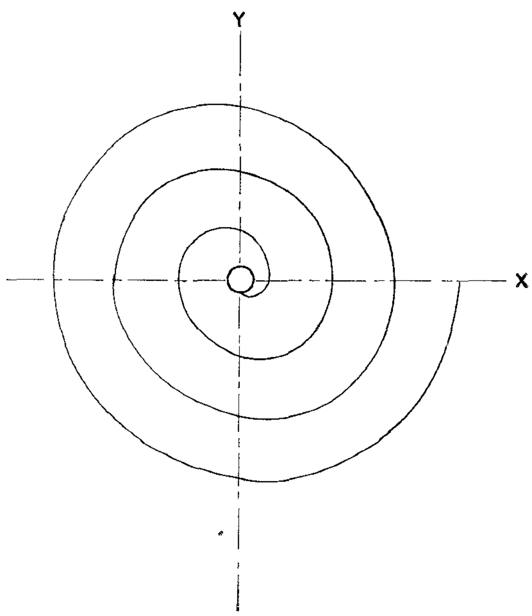

Fig. 3—Spiral wave scattered from thin cylinder. The spiral represents a line of constant phase.

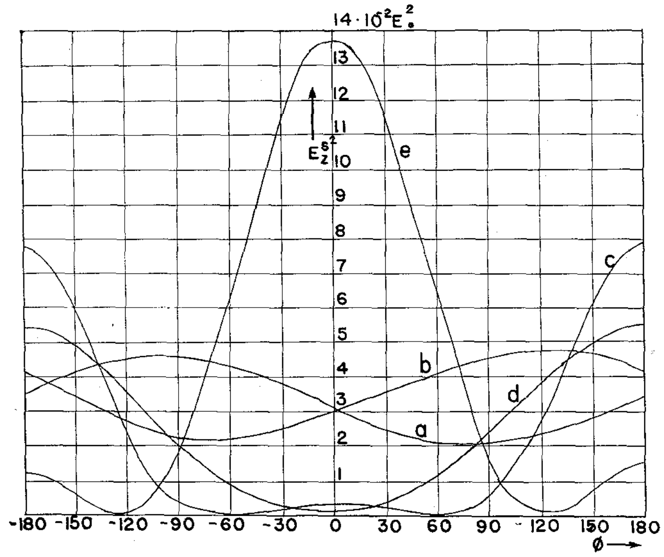

Fig. 4—Amplitude $(E^{scat})^2$ of scattered electric field as a function of scattering angle ϕ. (a) $\beta a=0.475$, $\beta_2 a=1.42$, $a=2.5\cdot 10^{-3}m$, $k/\mu=0.5$, $\beta r=20$. (b) $\beta a=0.475$, $\beta_2 a=1.42$, $a=2.5\cdot 10^{-3}m$, $k/\mu=3$, $\beta r=20$. (c) $\beta a=0.475$, $\beta_2 a=3.80$, $a=5.0\cdot 10^{-3}m$, $k/\mu=3$, $\beta r=20$. (d) $\beta a=0.475$, $\beta_2 a=1.42$, $a=2.5\cdot 10^{-3}m$, $k/\mu=0.25$, $\beta r=20$. (e) $\beta a=0.475$, $\beta_2 a=2.85$, $a=2.5\cdot 10^{-3}m$, $k/\mu=0.25$, $\beta r=20$.

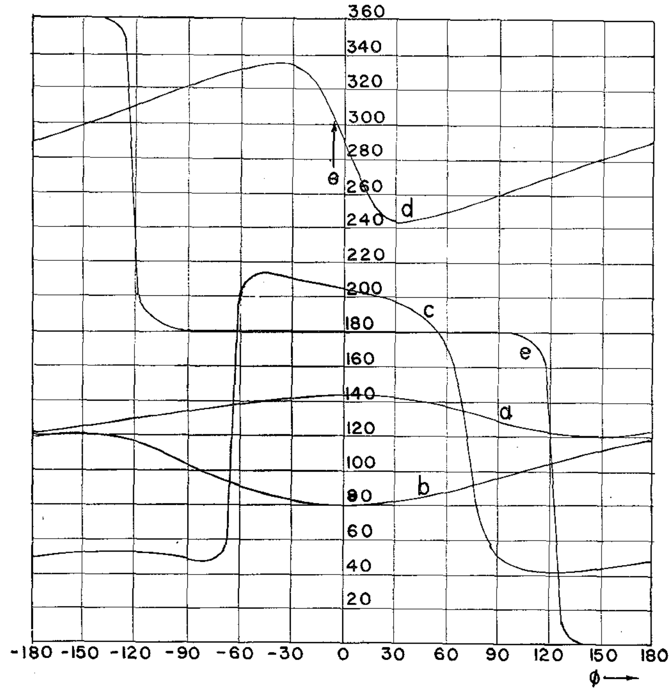

Fig. 5—Phase angle θ of scattered electric field as a function of scattering angle ϕ. (a) $\beta a=0.475$, $\beta_2 a=1.42$, $a=2.5\cdot 10^{-3}m$, $k/\mu=0.5$, $\beta r=20$. (b) $\beta a=0.475$, $\beta_2 a=1.42$, $a=2.5\cdot 10^{-3}m$, $k/\mu=3$, $\beta r=20$. (c) $\beta a=0.475$, $\beta_2 a=3.80$, $a=5.0\cdot 10^{-3}m$, $k/\mu=3$, $\beta r=20$. (d) $\beta a=0.475$, $\beta_2 a=1.42$, $a=2.5\cdot 10^{-3}m$, $k/\mu=0.25$, $\beta r=20$. (e) $\beta a=0.475$, $\beta_2 a=2.85$, $a=2.5\cdot 10^{-3}m$, $k/\mu=0.25$, $\beta r=20$.

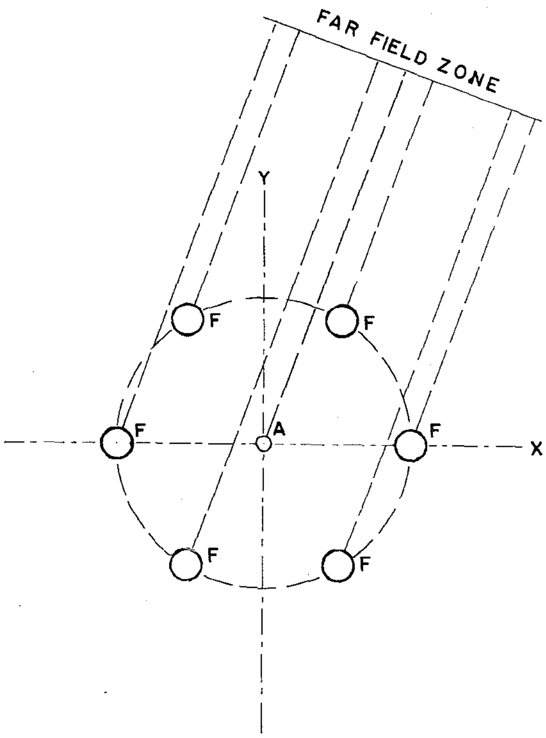

Fig. 6—Circular arrangement of six ferrite cylinders (F) around an antenna (A). The waves scattered from the cylinders are in phase with the incident wave from the antenna in the desired direction and 180° out of phase in the opposite direction.

Numerical calculations of (14) have been carried out on a Univac computer. In order to simplify the programming, the derivatives of cylinder functions and cylinder functions of negative order have been replaced by cylinder functions of positive order [see (31)].

The amplitude and phase of the electric field have been plotted in Figs. 4 and 5. For certain values of the parameters, a, k/μ, β_2 this exact solution yields also a spiral wave. It is interesting to note that except for high values of k/μ either the amplitude or the phase of the electric field is an odd function of the scattering angle ϕ, so that the scattered field is always asymmetrical.

Discussion and Conclusions

The foregoing discussion shows that the scattered field from a ferrite cylinder is in most cases asymmetrical about the direction of incidence. The direction of maximum field strength depends on the dc magnetization of the ferrite. By a suitable arrangement of several cylinders, as shown in Fig. 6, the scattered field can be concentrated in one direction. Here the phases of the scattered waves are in phase with the incident wave in the desired direction and 180° out of phase in the opposite direction. With a cyclic application of the magnetic dc field the field pattern is rotated. In order to distort the field as little as possible under this rotation a large number of scatterers should be used.

Because of the abrupt phase change for certain values of the deflection angle, it seems possible to obtain a narrow beam of a few degrees. The large variations of the amplitude, however, will result in strong sidelobes.

For the design of such an antenna the scattering pattern of the ferrite cylinder has to be measured. Then the time function of the magnetic dc field, which gives the desired antenna pattern, has to be determined.

The advantage of such an electronic scanning antenna is the lack of mechanical parts and the weightless rotation which allows much higher scanning speeds than with a mechanical system.

Acknowledgment

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Phase Adjustment Effects on Cascaded Reflex Klystron Amplifiers*

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Summary—**Reflex klystrons (type 2K25) were used as regenerative amplifiers for the X-band. Two 2K25 reflex klystron amplifiers were cascaded with a coupling circuit which contained a variable phase shifter. The effect of the phase adjustment was investigated in comparison with another coupling scheme which did not contain the phase shifter. The phase adjustment in the coupling circuit gave the amplifier system high gain (more than 50 db max.), and a reasonably low noise figure (8 db–17.5 db). High sensitivity was obtained. Proper phase adjustment of the two stage reflex klystron amplifier could give more than twice the gain in db of the single stage amplifier because of the regenerative feedback between stages. The linearity and dynamic range were considerably improved by the phase adjustment. But the frequency bandwidth became narrow (2 mc), and improvement in stability and directivity was not significant.**

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Introduction

It has been shown that ordinary reflex klystrons are usable as microwave regenerative amplifiers.[1–6] In order to obtain high gain, it is natural to think about cascading the reflex klystron amplifiers. This is, however, a rather complicated problem, because a part of the amplified power reflects back and forth between

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